The Relation between the Notions of Idealisation and Approximation in Science

DEMETRIS PORTIDES

University of Cyprus
portides@ucy.ac.cy

Abstract: The notions of 'idealisation' and 'approximation' are strongly linked to the question of "How our theories represent the phenomena in their scope?". Although there is no consensus on the nature of the process of idealisation and how it affects theoretical representation, at the level of science education we can still gain much from the insights of existing philosophical analyses. Traditionally, educational methodology treats the observed divergence between theoretical predictions and experimental data by appealing to the more common-sensical notion of 'approximation'. The use of the latter notion, however, to explicate discrepancies between theory and experiment obscures the theory/experiment relation. It does so, I argue, because 'approximation' either depends upon or it piggybacks on 'idealisation'. Thus 'idealisation' is a primary aspect of the theory/experiment relation, whereas 'approximation' depends on the former.

1. Introduction

That theoretical predictions and experimental measurements in science do not exactly match each other is commonplace. Had they matched there would not be much room for doubt that our theories gave us the truth about particular aspects of the world. The fact that they do not has led to philosophical debate on several related issues concerning our scientific enquiry. Some examples are: "Is it the approximate truth or the empirical adequacy of our theories that could be rationally justified?", "Do we have rational criteria of choice between competing theories?", "Given that our theories do not exactly mirror the world, how is it that they represent it?", etc. In this paper I shall explore the relation between 'approximation' and 'idealisation'. These two notions are features of the more general process of theoretical representation of physical systems and hence are strongly tied to the relation between theoretical statements and experimental reports. In fact, the discrepancy in the theory/experiment relation could be attributed to these two characteristics of scientific methodology in theory and model construction.

In science and to some extent in philosophy, attention is given primarily to the notion of approximation, possibly because of its more mundane nature but also because it is widely recognised that it can be explicated and handled exclusively by the use of mathematical tools. This has led to conceptual confusions firstly because the concept of idealisation and the process of idealisation in science by and large have been ignored, despite their epistemological and methodological significance, but also because it became customary to use the concept of approximation as a surrogate to idealisation, thus hindering the recognition of those elements of scientific practice that are associated with idealisation. That this indiscriminate use of approximation has created the impression that the two concepts could be used interchangeably is not an argument for the synonymy of the two concepts. I shall argue that the two concepts are in fact distinct, and that their interdependence is such that if clarified it could illuminate the theory/experiment relation.

As a first step in distinguishing the two concepts we could follow Suppe (1989) and understand *idealisation* of the features of physical systems to involve two primary modes: either (a) abstracting relevant features of the physical systems from the theoretical description, e.g. ignoring the effects of friction in the description of the motion of a body on an incline plane, or (b) distorting the characteristics of their relevant features, e.g. assuming a projectile to be a point-mass in estimating its trajectory.[1] *Approximation* of the features of physical systems could also be divided into two modes; it is achieved either (a) by simplifying the relevant parts of the descriptions of individual features and properties of the physical systems in the overall theoretical descriptions, e.g. assuming the effect of the damping force due to air resistance to the motion of the pendulum to be a linear or quadratic function of velocity, or (b) by simplifying the theoretical description of the physical system as a whole in order to produce a description that is not exact but it is tractable and close enough, e.g. assuming that the magnitude of all the effects to the motion of a body are small thus allowing us to ignore their mutual interactions and treat them as separate contributions that give rise to linearly independent tractable equations (later I shall demonstrate both of these modes of approximation with reference to the process of modelling the pendulum).

Aiming to minimise the epistemic effects of approximation, in science and mathematics much work is done in an attempt to minimize the theory/experiment discrepancy by the use of theories of systematic and random error of measurement. This however does not explicate the relation of approximation between theory and experiment, which is, it could be argued, a vague concept both in terms of its constitutive conceptual components and in regard to its impact on other semantic concepts like truth. Nevertheless, one thing is clear about the approximation relation; it is a concept that is closely linked to the concept of truth, and thus it is not surprising that recent philosophical attempts to explicate the discrepancy have focused, for instance, on a notion of approximation as degree of truth or truthlikeness (Popper 1979, 1989), despite the shortcomings of such an approach (see Psillos 1999).

It is a trivial matter that science is not concerned with the strict arithmetical sense of the notion of approximation. That is, the statement that the number *a* approximates the number *b* does not have any scientific significance. If a statement of such form is to have scientific value then the numbers *a* and *b* must be values of physical quantities that refer to the actual world. Hence the approximation relation between *a* and *b* takes on a different character when stated in a scientific context, it refers to the closeness of the values of two quantities. Not to any two unrelated quantities, but to quantities that purportedly refer to the same thing and which are computed in different ways, the first via the conceptual resources of a theory and the second via the experimental apparatus and the theories of measurement and experiment.[2] In short, the approximation relation refers to the closeness of theoretical predictions to experimental measurements, and this closeness has been traditionally interpreted by philosophers and scientists alike as closeness to truth, or truthlikeness, or verisimilitude of scientific theories. In the philosophical literature one can find several theories of approximation as theories of approximate truth (e.g. Niiniluoto 1987, 1999; Laymon, 1980, 1987), all of which have one common goal, to explicate what it means for a theoretical statement to be approximately true of the world. This task has not proved to be easy partly because of the vagueness of the concept of approximation and partly because of its relation and dependence on the process of idealisation in scientific methodology.

In science education it is tradition to ignore the vagueness of the approximation relation and in addition to take it as a primitive notion that suffices for the –approximate– truth or adequacy of the theory in deliberating upon the discrepancy in the theory/experiment relation. This practice not only does not clarify what it means for a certain theoretical prediction to approximate an experimental measurement, but it also obscures the theory/experiment relation and hence it leads to an elliptic understanding of the nature of scientific theories and models on behalf of the science student. In this paper I shall not attempt to explicate the concept of approximation but I will focus only on the second problem and argue that by hooking up approximation to idealisation we manage to maintain a more lucid view of the theory/experiment relation. In fact, I will motivate a much stronger thesis that a clear understanding of the ways by which idealisation and approximation interrelate is necessary for explicating the theory/experiment relation.

2. Distinguishing Idealisation from Approximation

By hooking up the two concepts it should not be understood to mean that they are indistinct. On the contrary, they are clearly distinct concepts and this can be seen by inspecting their logical properties. We generally understand idealisation, but not approximation, as a directional process. This intuition is captured by the logical property of symmetry. Idealisation is not a symmetric concept (in fact, it could be claimed that it is asymmetric) whereas approximation is. That is to say, thinking of idealisation and approximation as relations in which two statements enter (e.g. one deriving from theory, X, and the other from experimental reports, Y), if "X is an idealised description of Y" is true then it is not true that "Y is an idealised description of X", whilst if "X is an approximate description of Y" is true then it is also true that "Y is an approximate description of X". For example, the simple harmonic oscillator exemplifies what we would consider to be an idealised description of a target physical system like the motion of the pendulum in the lab; however, a description of the pendulum that accounts for all factors influencing the motion of the bob is not an idealised description of the simple harmonic oscillator. On the other hand, if the simple harmonic oscillator predicts that the earth's acceleration g is equal to $9.8 m/s^2$ and this is accepted to approximate the value of $9.81 m/s^2$ that results from measurements on the pendulum motion, it makes equally good sense to claim the converse that $9.81 m/s^2$ approximates the theoretical prediction of $9.8 m/s^2$.

In addition to being asymmetric idealisation is transitive, i.e. if "X is an idealised description of Y" and "Y is an idealised description of Z" then "X is an idealised description of Z". This property captures our intuition that idealisation is a scalable concept, which is demonstrated by examples like the following: if the simple harmonic oscillator is an idealised description of the damped harmonic oscillator and the latter is an idealised description of the pendulum then the simple harmonic oscillator is also an idealised description of the pendulum. Approximation on the other hand is not –unconditionally– transitive since it is a pragmatic –and context-dependent– issue whether, if "X is an approximate description of Y" and "Y is an approximate description of Z" then "X is an approximate description of Z", and hence the conditional is not true for all X, Y and Z. This logical difference is in a sense indicative of the fact that the two concepts differ in their pragmatic components (e.g. their role in heuristics), an issue to which I shall return in the last section of this paper. Furthermore, another characteristic that distinguishes the two concepts is that approximation is understood as having end-point limits, whereas idealisation cannot have clearly-cut limits. This intuition is partially captured by the property of reflexivity. Idealisation is not reflexive (and it could also be argued that it is irreflexive), i.e. it makes no sense to say that X is an idealisation of itself unless the concept is trivialized, whereas approximation is reflexive, i.e. the statement "X is equal to itself" can be understood to mean the limiting case of the approximation relation.

With these logical properties of the two concepts in mind it is clear that aphorisms like "all idealisations are forms of approximation" or "all approximations are forms of idealisation", which one encounters in philosophical as well as scientific literature, either distort our intuitions of the two notions or require careful qualification if they are to shed light on the theory/experiment relation. For the science educator the problem is twofold, on the one hand there are two concepts that are related in not very obvious ways and whose distinction is subtle and not easily comprehensible to the science neophyte. On the other hand, there are two methodological processes in science, that are equally important for the best possible understanding of science, that are closely related to each other but which if not discerned properly the theory/experiment relation is obscured; moreover, despite the fact that the concepts of idealisation and approximation are logically distinct, mere inspection of actual science reveals that the scientific processes in which they are employed are interconnected.

Looking at idealisation and approximation from a methodological perspective another difference can be discerned that can be located in how the two concepts are employed in scientific representation. It is widely admitted that one of the functions of idealisation is to reach a level of generality in our representations of phenomena. When it is claimed that "X is an idealised description of Y" it is implied that Y is not necessarily a description of a token case. The simple harmonic oscillator, for instance, is an idealised description of the type 'pendulum' and not just of particular token pendulums. The same also holds for different levels (or degrees) of idealisation. The somewhat de-idealised version of the simple harmonic oscillator, known as the damped harmonic oscillator, also represents in a general way even though the class of physical systems it represents is not necessarily the same as that of its more idealised predecessor model. The rationale behind representation by the use of the idealisation/de-idealisation process is that the more idealised the representational model is the less properties and features of the type target systems it represents. By de-idealising a model we do not restrict the class of representations, but we add more of the relevant features in the general representation of the type target system. The process of idealisation is therefore used for general as well as for particular representation. This characteristic of representation is not something that is present in approximation claims. We cannot, for instance, claim that the simple harmonic oscillator represents the pendulum in general because it *approximates* the type pendulum, since there are actual pendulums with very large damping forces that make them anything but approximate to the model. In short, approximation is a feature of representation that concerns the specific or the token cases of target systems. This is another reason why not all idealisations are approximations, although the converse may be the case. More importantly, however, in order for an approximation claim, i.e. "X approximates Y", to be useful in the explication of the theory/experiment relation, X must be such as to refer to a token physical system that involves a large enough number of features and properties that are present in Y. Otherwise X and Y would not be referring to the same thing. In other words, the statement "X approximates Y" is useful for explicating the theory/experiment relation if X is sufficiently de-idealised so that it can be regarded as a genuine representation of Y. Another way of saying this is that X must be sufficiently de-idealised so that its reference is no longer a class of ideal type systems, but types that can be actualised in the world. Because of this I claim that the process of idealisation is primary and upon it the pragmatics of the process of approximation depend (e.g. the role and use of approximation in representing physical systems, or the factors that are responsible for the successful use of approximation in prediction and explanation, etc.).

My argument is backed by an analysis of the well-known model of the simple harmonic oscillator (of classical mechanics) and the process by which it is used in order to construct derivative models that can be proposed for the representation of target physical systems such as the simple pendulum or the torsion pendulum. I employ this analysis of the process of construction of representational models to demonstrate that idealisation, and its converse process of de-idealisation, is present at every level of scientific theorizing whereas the concept of approximation becomes methodologically valuable, and epistemically significant, after a certain point in the process is reached when a given theoretical construct (i.e. a scientific model) may be proposed for the representation of a physical system. Thus I explicate the dependence of approximation on idealisation on pragmatic grounds. In other words, although both concepts are epistemic in nature scientific methodology requires that a process of de-idealisation takes place before we can meaningfully employ the notion of approximation. Hence idealisation is a primary process in our scientific methodology and approximation piggybacks on it. Thus, science education must accommodate this result in the analysis of the theory/experiment relation if the goal is to elucidate the latter.

3. Linking Approximation to Idealisation

Although it is not the purpose of this paper to offer a theory of approximation that would clarify the notion, some aspects of the use of the concept are worth clarifying since my concern here is the use of the concept in illuminating the theory/experiment relation and hence it is crucial that its relation with idealisation is understood. The ambiguity present in the statement that our theories approximate the world partly has to do with what the statement refers to and at what level of discourse it is used. Sometimes the notion of approximation is used as part of a meta-meta-scientific statement. Such is the case when it is claimed that the idealised description of the simple harmonic oscillator approximates the motion of the torsion pendulum, or more generally when it is claimed that *idealised* descriptions of physical systems *approximate* their actual target physical systems. In philosophical discussions of idealisation, this use of the notion of approximation is in a sense present in the view that good idealisations are distinguished from bad ones if their claims approximate the world. In such uses, the reference of approximation is either to the degree or to the kind of idealisation and not to the actual relation between the theoretical and experimental statements. A meta-meta-scientific use of the concept is employed to qualify characterisations of scientific statements and their relation to experiment hence it must be discerned from its meta-scientific use. Meta-scientific use means that approximation itself is a characterisation of scientific statements and their relation to experiment. The focus in this paper is to the latter use of approximation, which I believe to be the epistemically important use of the concept. At the meta-scientific level of discourse we could make either of two kinds of approximation claims. We could claim that X approximates –the truth about– Y, when X and Y describe properties and processes and the descriptions of X closely resemble those of Y. We could also claim that X approximates Y when X and Y are real-valued functions and the value of X is close to the value of Y for particular values of their arguments. Or we could claim that approximation refers to a combination of both of the above.

The first kind of approximation claim can easily be changed over to an idealisation claim of either of the two modes mentioned earlier, i.e. if X approximates Y, then X is an idealised description of the properties and processes of Y, either because relevant features of Y have been abstracted from X or because the characteristics of relevant features of Y have been distorted in X. The connection between approximation in this sense and idealisation can easily be seen because when we identify approximation with closeness of resemblance of the properties and processes in two descriptions it is either because some of the characteristics of Y are absent from X or because some of the characteristics of Y have been changed or distorted in X or because of both reasons. In this sense approximation is used as a surrogate to idealisation (and coincides with the latter's meaning) and adds nothing more to the content of the characterisation of the relation between the statements X and Y that idealisation would not. Because approximation in this sense is understood as being proportional, so to speak, to the number of features that have been abstracted or distorted in the theoretical description, often one is led to the view that a description X approximates Y better than Z does only if it is less idealised than Z; but this way of linking approximation to idealisation is unnecessary since it does not add anything instructive to the relation between the two concepts because in this sense all approximations are specific forms of idealisation. Nevertheless the distinction between the two concepts is still useful because idealisation is a much wider concept and not every idealisation is an approximation even in this sense, as I have argued above.

The second kind of approximation claim presents a more complicated problem. Clearly this kind of approximation claim is distinct from the notion of idealisation and very hard to relate to the latter. Because it is a direct consequence of representing theoretical descriptions in mathematical languages approximation in this sense seems to be a concept that could be explicated exclusively by mathematical considerations. Because of this some philosophers (e.g. Laymon, 1980, 1985, 1987) have attempted to relate idealisation to approximation by also explicating the former primarily in terms of mathematical considerations. Such attempts, however, fail to achieve a full explication of the process of idealisation in science because as a conceptual process idealisation is not a characteristic restricted to mathematical languages alone.

I shall herein concentrate on yet another problem with this view. If we do understand the statement that "our theories approximate the world" as referring to a relation that can be explicated exclusively by mathematical considerations then we are faced with the following two possible points of view of approximation pointed out by Redhead (1980). The first is approximate solutions to exact equations. Consider his example: 'For the equation $dy/dx - \lambda y = 0$ we might expand our solution as a perturbation series in λ, the n^{th} order approximation being just $y_n = 1 + \lambda x + \lambda^2 x^2/2! + ... + \lambda^{n-1} x^{n-1}/(n-1)!$, if we consider the boundary condition y=1 at x=0.'(*Ibid*. p. 150) The second view of approximation that Redhead calls to mind is when we look for exact solutions to approximate or simplified equations. In the example above, y_n is an exact solution to the equation $dy/dx - \lambda y + \lambda^n x^{n-1}/(n-1)! = 0$, which for small λ is approximately the same as the original equation above. It is easy to prove, as Redhead indicates, that the two views are equivalent since, '…if we consider an approximate solution y_n for an exact [equation] …we can always specify [another equation] …which is 'approximately' the same as the first, for which y_n is an exact solution.' (*Ibid*. p. 150) Now, the number of logically possible approximate solutions to an exact equation is infinite and each of these is an exact solution to another equation which is an approximate or simplified version of the exact equation. Thus by viewing approximation only in a mathematical sense we run into the problem that different equally plausible approximating equations that purportedly represent the same target physical system will yield somewhat different solutions that will not be experimentally distinguishable. Hence it follows that we would have no non-arbitrary way of singling out one solution that approximates the data and represents the corresponding physical system if we focus only on mathematical considerations. There are various ways to see the consequences of this problem. One simple way, for instance, would be to suppose that we have a choice between two theoretical constructs that are meant to represent a particular physical system (e.g. a pendulum) such as the models of the simple harmonic oscillator and the damped harmonic oscillator. For the sake of the argument, let us suppose that the first model predicts a period of oscillation equal to a and the second predicts a value equal to b, now suppose that the measurement of the period of oscillation of the pendulum is such that it approximates both predictions without distinguishing between them (e.g. $(a+b)/2$). Based merely upon the criterion of approximation (understood only in mathematical terms) in choosing the correct representation of the physical system means that we cannot choose between the two in a non-arbitrary way. This may seem as a very simple example, with which we are so familiar that we are tempted to say that the damped harmonic oscillator is a better choice for modelling the pendulum because had experimental inaccuracies not been present experiment would have distinguished the two, since we know that there is in fact a damping force acting on the real pendulum. This response however is not convincing because we can imagine encountering the same problem in modelling a system we are not familiar with, in which case we are not familiar with the factors that influence the physical quantity in question. With this in mind we are led to the conclusion that approximation of the experimental value by the theoretical prediction is not a sufficient condition for proximity to truth, but also neither is it the way scientists go about in choosing their theoretical representations. The way around this, I suggest, is to link approximation to the process of idealisation on pragmatic and methodological grounds so that non-mathematical considerations also become part of our explication of the concept of approximation and subsequently of the theory/experiment relation.

4. The Interplay between Idealisation and Approximation in Modelling the Pendulum

The process of idealisation enters at different levels of scientific theorising. Two principal levels could be identified that are useful to our understanding of how theories are formulated and applied. Assuming that we begin with the universe of discourse, the first level of idealisation that could be distinguished is that of selecting a small number of variables and parameters abstracted from the phenomena and used to characterise the general laws of a theory. For example, in classical mechanics *position* and *momentum* are selected and used to establish a relation which we call Newton's 2^{nd} law or Hamilton's equations. By abstracting a set of parameters we thus create a sub-domain of the universe of discourse in which the scope of the theory is confined and which we call the domain of a scientific theory. Thus, Newton's laws signify a conceptual object of study that we may call the domain of classical mechanics; similarly the Schrödinger equation signifies the domain of quantum theory, and so forth. Scientific domains, viewed from this perspective, are clearly distinct from physical domains, which they could represent only if they are expanded by or integrated with other conceptual resources. For instance, the dynamics of bodies may be influenced by factors that are related to electrical or heat phenomena that are not accounted by Newton's laws. In all the laws (which we may call idealised, in the sense that they are established by a small number of abstracted parameters) something is left unspecified: the force function in Newton's 2^{nd} law, and the Hamiltonian operator in the Schrödinger equation. The specification of these is what would establish the link between the assertions of the theory and physical systems. The description I propose of this level of idealisation in scientific theorising is similar, if not identical, to Suppe's version of the Semantic View (Suppe 1974, 1989), where he maintains that by abstracting a small number of variables and parameters in order to characterise the general laws of a theory we thereby define a class of mathematical structures or models that may be used for the representation of phenomena.[3] It is evident that the notion of approximation does not enter at this level of theorising.

The second principal level in which the process of idealisation enters in our scientific theorising is the process of specifying force functions or Hamiltonian operators etc. and it is effective in allowing us to bridge the assertions of the theory to physical systems. At this level, the process of idealisation is intertwined with that of approximation and in what follows I shall demonstrate this process and attempt to show the pragmatic nature of the relation between the two by analysing how scientists model the simple pendulum.

Morrison (1999) has argued that in order for an idealised model, such as the simple harmonic oscillator, to accurately represent the respective physical system we cannot rely on theory alone but we must add several correction factors to the model. In order to analyse the process of constructing a representational model of the pendulum by blending a theoretical model with the relevant correction factors, it will be helpful if we work with a distinction between two kinds of model that I shall label the ideal model ($model_I$) and the concrete model ($model_C$). Let the class of ideal models be the class of theoretical models (as understood by the proponents of the Semantic View, e.g. Gierre 1988; van Fraassen 1980, 1989; Suppe 1974, 1989; da Costa, N. C.A. and French, S. 1990, 2003)[4] augmented with the class of models that –unproblematically– could be understood as mathematical approximations of the former. Let the class of concrete models be the class of those models that are proposed by scientists for the theoretical representation of physical systems. Distinguishing between $model_I$ and $model_C$ is not meant to mark a separation between theoretical and *a posteriori* models. $Model_I$ is the theoretical model that we initially attempt to fit the physical system into, however its representational capacity is only –to say the most– suggestive. We could regard model$_C$, on the other hand, as the carrier of all the antecedent knowledge and physical intuitions that direct us to capture in concrete ways the attributes and features of a particular physical system. My thesis is that to turn a model$_I$ into a representation of a physical system we must blend it with conceptual resources that extend beyond the conceptual confines of the theory and in the process the result is a distinct entity that I call a model$_C$. The distinction is therefore not based on mathematical tractability but it is used primarily to emphasise the fact that the conceptual resources of model$_I$ are confined to the theory that gives rise to it, whereas those of model$_C$ extend beyond the theory.

Frequently in classical particle mechanics the initial stages of modelling a physical system involves the employment of one of the available models$_I$. The process by which the model$_I$ is chosen and employed has been analysed by Cartwright (1983) and dubbed as 'theory entry'. The 'fitting of facts to equations', Cartwright suggests, is a process that can be divided into two stages. As a first stage we prepare an informal description of the phenomenon such as to '…present the phenomenon in a way that will bring it into the theory.' (Cartwright, 1983, p. 133) In this stage we use our background knowledge and try to confine the description to those elements that will allow us to match an equation to the behaviour of the physical system. In the second stage, we look at the description through the prism of the theory and dictate the necessary equations, boundary conditions and approximation methods. In the context of my discussion, the process of theory entry is important because in preparing a description of the phenomenon as to bring it into the theory we most frequently distort some features of the phenomenon or abstract others. Theory entry thus opens up the scene for a third stage which runs parallel to the second and which is operative in theory-application: the informal descriptions of the phenomena act as guidelines for the corrections that should follow the process of theory entry. This stage leads to the construction of a representational model of the target physical system and thus a relation between theory and experiment is established. The process involves the 'moulding' of the equations of the model$_I$ as to capture as many of the features of the physical system as possible and the result is a model$_C$.

To achieve theory entry for the pendulum we begin with a highly idealised description of the phenomenon that would sanction the use of a model$_I$. By assuming a mass-point bob supported by a massless inextensible cord of length l performing infinitesimal oscillations θ about an equilibrium point, the equation of motion of the simple harmonic oscillator (i.e. a model$_I$) can be used as the starting point for modelling a real pendulum and thus attempting to measure the acceleration due to the Earth's gravitational field:

$$\ddot{\theta} + (g/l)\theta = 0 \qquad (1)$$

The solution of this equation yields a relation among the period T_o, the cord length l and the acceleration g due to the Earth's gravity: $g = 4\pi^2 l / T_o^2$

The experimental problem of determining g, therefore comes down to measuring l and T_o. However, T_o is far from an acceptable level of accuracy to the experimental value of the period T. This is expected, because it is known that the actual pendulum apparatus is subject to influences that are not accounted for in the idealised assumptions underlying equation (1). That is to say, the model$_I$, expressed through equation (1), involves many abstractions and idealisations that minimise its representational capacity. In fact I encourage an even stronger claim, that the model$_I$ does not refer to the class of actual pendulums but to a class of ideal types that cannot be actualised, i.e. the class of mass-point bobs supported by a massless inextensible cord performing infinitesimal oscillations about an equilibrium point. Hence to claim that equation (1) describes approximately the motion of the pendulum is to commit an error in the reference of the model, since it does not refer to the pendulum but to a class of ideal-types that may resemble in some respects the characteristics of actual pendulums. Moreover, the kind of representation we could have in the relation between model$_I$ and the pendulum is a type-type kind and to attribute to such a relation the characteristic of approximation is erroneous since, as explained earlier, the approximation relation is an attribute of token-token cases. Furthermore, if we were to claim that equation (1) describes approximately the motion of the pendulum it would be equivalent to claiming that T_o approximates T, but this would lead us to the problem of non-arbitrary criterion for choice, explained earlier. That is the solution T_o of equation (1) is experimentally indistinguishable from many other possible solutions that are equally good approximations to T, hence the relation of approximation does not offer a non-arbitrary criterion of choosing the correct representation. Hence, even if T_o and T numerically approximate each other it would not mean that the truth about the physical system is approximated by the model$_I$. But we must also recognise that the reason physicists expect the two values to differ significantly is because they know that a large number of important influencing factors are not included in the theoretical description.[5] My contention is that when the degree of idealisation is high such that the theoretical construct refers to a class of ideal-types the concept of approximation cannot be employed in any scientifically instructive way; hence we must search elsewhere in order to illuminate the theory/experiment relation.

In their attempt to construct a representational model of the pendulum, Nelson and Olsson (1986) give the following list of influencing factors, that model$_I$ does not account for: (i) finite amplitude, (ii) finite radius of bob, (iii) mass of ring, (iv) mass of cap, (v) mass of cap screw, (vi) mass of wire, (vii) flexibility of wire, (viii) rotation of bob, (ix) double pendulum, (x) buoyancy, (xi) linear damping, (xii) quadratic damping, (xiii) decay of finite amplitude, (xiv) added mass, (xv) stretching of wire, (xvi) motion of support. They proceed to show how the value T_o can be corrected by introducing the different correction factors into the equation of motion. In effect, they are attempting to show what is involved and how it is involved in the construction of a model$_C$ that can be used for the theoretical representation of the actual pendulum apparatus. Consider some of the examples analysed by Nelson and Olsson (1986):

(1) Since the pendulum experiment takes place in air, it is expected that by Archimedes' principle the weight of the bob will be reduced by the weight of the displaced air. Since under such circumstances the effective gravity is reduced, this increases the period. The correction factor is determined by accounting for the mass of the air displaced.

(2) The air resistance acts on the oscillating system (pendulum bob and wire) to cause the amplitude to decrease with time and to increase the period. The Reynolds number for each component of the system determines the law of force for that component. The drag force is hence expressed in terms of a dimensionless drag coefficient, which is a function of the Reynolds number. In the pendulum case it can be shown that a quadratic force law should apply for the pendulum bob, whereas a linear force law should apply for the pendulum wire. Hence, it makes sense to establish a damping force which is a combination of linear and quadratic velocity terms: $F = b|v| + cv^2$. To determine the physical damping constants b and c the work-energy theorem is employed, an appropriate velocity function $v=f(\theta_o,t)$ is assumed, and under the assumption of conservation of energy they are matched to experimental results. They proceed to solve the resulting equation of motion and determine the correction factors.

(3) A real pendulum has a bob of finite size, a suspension wire of finite mass and in addition the wire connections to the bob and the support have structure. All these factors have some contribution to the oscillations. Their effects are incorporated into the physical pendulum equation: $T = 2\pi\sqrt{I/Mgh}$. Where, I is the total moment of inertia about the axis of rotation, M is the total mass and h is the distance between the axis and the centre of mass.

Depending on the shape of the bob we could calculate its moment of inertia and thus compute its contribution to the period of oscillation. Nelson and Olsson (1986) assume that the bob is a perfect sphere of radius *a* and proceed to compute a correction to the period. In a similar manner the correction contributions due to the wire connections and the mass and flexibility of the wire are computed.

(4) The length of the pendulum is increased by stretching of the wire due to the weight of the bob. By Hooke's law, when the pendulum is suspended in a static position the increase is $\Delta l = mgl_o / ES$, where S is the cross-sectional area and E is the elastic modulus. The dynamic stretching when the pendulum is oscillating is due to the apparent centrifugal and Coriolis forces acting on the bob during the motion. This feature is modelled by analogy with the spring-pendulum system to the near stiff limit. The result is a system of coupled equations of motion, which when solved yields the correction factor for the period.

These examples indicate a number of complexities involved in the process of constructing model$_C$. The root of these could be traced in the attempt to relax or overcome the underlying idealisations and abstractions of model$_I$, which could be put in the language of physicists: when the goal is to model a physical system then the initial problem of starting with a law of force (i.e. Newton's 2nd law) and using it to find a model$_I$ for the description of the physical system does not suffice. In this quest, the general problem of finding the law of force that may be responsible for a particular constituent of the external force function in Newton's law, and which would reduce the degree of idealisation, is of equal importance. In order to determine the various force laws to be used in model$_C$ we utilise either the antecedently available empirical laws (such as Archimedes' principle, the Reynolds number and the drag force expression, and Hooke's law, for the case of the pendulum) or postulate novel physical mechanisms. By employing the various force laws in the construction of the model$_C$ we are turning the model into a representation of the respective physical system because when these correction factors are added the reference of the model is no longer a class of ideal-types but a class of actualisable systems. To consider that a model$_C$ approximates the physical system is not only reasonable but also scientifically useful because all these factors are approximations to particular aspects of the target physical system.

The construction procedure of model$_C$ is conventional and not peculiar to the pendulum. The mathematical functions for each influencing factor are determined by the use of various empirical theories from disparate areas of physics and are inserted into the equation of motion in a cumulative manner. Because the influence of each of these factors on the system is small, it is assumed that the resulting equation of motion approximates a system of linearly independent differential equations, each involving a different influencing factor. Each of the equations is solved individually to determine the values of the individual effects and the total value of the correction is computed by adding all the effects linearly (see Nelson and Olsson 1986). The methodological process we are faced with is the blending of experimental parameters and empirically determined laws together with a theoretical model to produce a model$_C$. The theoretical model is a pure derivative of the theory that we can turn into a *representation* of a physical system by blending it with these ingredients. This is done in an effort to extend the scope of application of the theory beyond the class of ideal-type systems (e.g. isolated point-masses and inelastic cords) to which the class of theoretical models may be understood to refer. To achieve this we give a concrete and specific context to the force function (i.e. to the abstract concept of 'force') for each and every different influencing factor. It is important to note that de-idealisation is the process by which the model is turned into a representation of the physical system. Approximation is the process by which the equation of the model$_C$ is made tractable. Both processes are in a constant interplay in trying to turn a model$_I$ into a model$_C$ but the epistemic significance of approximation depends upon the degree of de-idealisation achieved and this is the pragmatic aspect of the relation between the two concepts.

In the process of constructing model$_C$ above the primary concern is to discover those correction factors that would bridge the gap between model$_I$ and the target physical system, at this stage only the de-idealisation process is operative. The two modes of the approximation process enter into the picture once the de-idealisation process begins. When each correction factor, and the force law responsible for its behaviour, is discovered it is approximated by a mathematical expression that gives rise to a tractable equation of motion. This part of the process is an example of the first mode of approximation which clearly piggybacks on the de-idealisation process. Once all the correction factors are introduced into the equation of motion, i.e. when the process of de-idealisation is completed and the model$_C$ is constructed, the second mode of approximation is used. The assumption that the effects of all correction factors are small hence we could approximate the equation of motion with a system of linearly independent tractable equations also piggybacks on the de-idealisation process, in the sense that the de-idealising assumptions dictate the approximation techniques to be used.

In modelling physical systems the starting point is an idealised model, such as the harmonic oscillator, whose force function could be expressed through a general functional relation: $H(x) = f_1(g_1(x), ... ,g_n(x))$. A first-step de-idealisation would be to expand the functional relation by accounting for an influencing factor that has been initially ignored. This results in a new general functional relation which in its most simplified logical form could be presented as follows: $H'(x) = f_1(g_1(x), ... ,g_n(x)) + f_2(h_1(x), ... ,h_m(x))$. Supplementing the function $H(x)$ with cumulative correction factors is a process that goes on until our conceptual resources and background knowledge are exhausted. The idealised model can be understood to relate to its derivative de-idealised relatives in the following general way: $\lim_{f_2 \to 0}\left(H'(x) = f_1(g_1(x), ... ,g_n(x)) + f_2(h_1(x), ... ,h_m(x))\right) = H(x)$. In other words, on this account 'idealisation' is the process by which we let factors that are influential to the physical system tend to zero. De-idealisation is the converse process of allowing these factors to take finite values. That is, idealisation ignores the influence of factors and de-idealisation reintroduces their effect into the model. Approximation enters into this picture because each f_i is represented via the mathematical language of the theory in an approximate way and because the final H^k is solved by an appropriate approximation technique.

This process could be misconceived to mean only that model$_I$ is a description with some unspecified parameters and the model$_C$ is a description with those parameters specified, thus the latter is a structure-type nested in the former. In other words, by specifying parameters we effectively create a sequence of nested mathematical structures. The idealisation/de-idealisation process viewed from this perspective is no more than a partial ordering of structures. The criterion (i.e. relation) of this partial ordering is that of the restriction of the domain, i.e. two models, M_1 and M_2, are partially ordered if and only if the domain of M_2 is a restriction of the domain of M_1. We could think of the criterion for partial ordering as a transformation rule that requires the specification (or addition) of a parameter in the above functional relation. In this picture the reference of the sequence of models remains constant, i.e. the reference of model$_I$ would not be different from the reference of model$_C$, other than the restriction of the domain. Hence in this understanding of idealisation the use of approximation is meaningful at every level. However, the misconception in this view of idealisation is that by specifying a parameter we are not simply correcting our mathematical description $H(x)$ but we are bringing our theory in touch with the world, i.e. f_1 derives from the theory alone but f_2 derives from empirical laws and experimental parameters. So de-idealisation is not just a process by which we paste together different descriptions to create a more complex final description, but it is the process by which we supply a theoretical description, that refers to a class of ideal-types, with those conceptual ingredients that would make it refer to actual physical systems (i.e. in the words I have chosen to present it in this paper, it is the process of turning a model$_I$ into a model$_C$). If idealisation were understood in the former way then approximation would be suitable at every level. But if idealisation is understood as I suggest then approximation is useful only when we have turned a model$_I$ into a model$_C$. This is what I mean when I claim that the pragmatics of approximation depend upon idealisation. Unless a model$_C$ is established by means of de-idealising techniques and hence a plausible representation of a target physical system is constructed we cannot employ the relation of approximation without obscuring the theory/experiment relation.

5. Conclusion

The idea that theories do not represent the concrete circumstances in which naturally occurring physical systems are found was pointed out by several authors (e.g. Cartwright 1983, 1999; Shapere 1984; McMullin 1985; Laymon 1985; Morrison 1998). Among them proponents of the Semantic View like Suppe (1989) well-understand that theoretical models are abstract and idealised descriptions and as such, it could be claimed, they do not represent physical systems in any direct sense. My argument leads to the contention that it is only after they give rise to a model$_C$, appropriate for the representation of a particular physical system, that they acquire a certain capacity of representation. We say that the linear harmonic oscillator approximately represents the simple pendulum system, only because we have managed to use it successfully to construct a model$_C$. We would not claim that all conceivable theoretical models are representations of physical systems. But what is more important, in the context of my discussion, is that a model$_C$ is a representation of a target physical system that involves a theory derived description blended with empirical laws and other auxiliaries, and this is why it makes sense to call it an approximation of the corresponding physical system. Theoretical models refer to a class of ideal types whose empirical content is supplied when they are used in the construction of a model$_C$, by de-idealising them we change the reference class to actualisable physical systems and thus we can meaningfully employ the notion of approximation. Given the picture of scientific modelling that I have drawn and given the arguments that I have given of the difference between the concepts of idealisation and approximation, one consequence is that science education must give the necessary weight to the processes of idealisation and approximation if the student is to be come to terms with the complexities of the theory/experiment relation.

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[1] Suppe (1989) refers to the first mode as abstraction and to the second mode as idealisation. For these terminological distinctions see also Cartwright (1989), Morrison (1999), Portides (forthcoming a). For the purposes of this paper this terminological distinction will be ignored. McMullin (1985) also calls something like the first mode ‘material’ and something like the second mode ‘formal’ idealisation, both of which he places under the more general category of ‘construct idealisation’, where the latter is distinguished from ‘causal idealisation’.

[2] A concern in Science and Philosophy of Science is the question whether two competing theories e.g. Relativity Theory and Newtonian Mechanics, stand in the relation of approximation to each other. Whether it is reasonable to view Newtonian Mechanics as an approximation to Relativity theory at some limit is an issue that concerns the relation between two mathematical calculi and their interpretation and not the theory/experiment relation in any direct sense. In this paper I explore the notion of approximation as a relation between theory and experiment and not as an intertheoretic relation. I do believe, however, that my argument could be generalised as to accommodate the later use of the notion.

[3] Elsewhere (Portides, forthcoming b) I have disputed the contention that such models of the theory could in fact be used for the representation of phenomena without being integrated with conceptual resources that transcend the theory’s apparatus, but in this paper my concern is different and I shall avoid this issue.

[4] According to the Semantic View, the class of *theoretical models* could be defined by the laws of the theory. E.g. in classical mechanics by means of the *position* and *momentum* vectors we establish a relation: Newton's 2nd law. The specification of any force function would define a theoretical model. For instance, if the force function is specified as $F=-k\xi$ (for a position coordinate ξ and constant parameter k), then the 2nd law defines such a model (known as the linear harmonic oscillator) that is expressed by the equation of motion: $\xi''+(k/m)\xi=0$. If the force function is specified as $F=-k\xi+b\xi'$, then the 2nd law defines another such model (known as the damped harmonic oscillator) expressed through the equation of motion: $\xi''-(b/m)\xi'+(k/m)\xi=0$, and so on. The mathematical structure of the theory, defined by the position and momentum vectors related through Newton's 2nd law, thus lays down an indefinite number of possible theoretical models which are available for representing mechanical systems. Notice that I draw a distinction between theoretical models and representational models and upon it my argument rests.

[5] This is one way to understand why when it is not possible to determine scalable de-idealised versions of a $model_I$ physicists employ perturbation theory (particularly in Quantum Mechanics), which is roughly a way to represent the aggregate effect of the different factors that influence the system. In other words perturbation theory is a way to de-idealise and approximate simultaneously by representing an aggregate effect rather than the individual effects of each influencing factor. But when perturbation theory is employed physicists are not suggesting that the highly idealised $model_I$ approximates a physical system, but that the $model_I$ supplemented by an approximate representation of the aggregate effect of influencing factors approximates the physical system. In other words by adding the perturbation term the reference of the model in assumed to have changed.